

Time Reversal Signal Processing in Communications-A Feasibility Study

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1. Introduction

A typical communications channel is subjected to a variety of signal distortions, including multipath, that corrupt the information being transmitted and reduce the effective channel capacity. The mitigation of the multipath interference component is an ongoing concern for communication systems operating in complex environments such as might be experienced inside buildings, urban environments, and hilly or heavily wooded areas. Communications between mobile units and distributed sensors, so important to national security, are dependent upon flawless conveyance of information in complex environments. The reduction of this multipath corruption necessitates better channel equalization, i.e., the removal of channel distortion to extract the transmitted information. But, the current state of the art in channel equalization either requires a priori knowledge of the channel or the use of a known training sequence and adaptive filtering. If the "assumed" model within the equalization processor does not at least capture the dominant characteristics of the channel, then the received information may still be highly distorted and possibly useless. Also, the processing required for classical equalization is demanding in computational resources. To remedy this situation, many techniques have been investigated to replace classical equalization. Such a technique, the subject of this feasibility study, is Time Reversal Signal Processing (TRSP).

Multipath is particularly insidious and a major factor in the deterioration of communication channels. Unlike most other characteristics that corrupt a communications channel, the detrimental effects of multipath cannot be overcome by merely increasing the transmitted power. Although the power in a signal diminishes as a function of the distance between the transmitter and receiver, multipath further degrades a signal by creating destructive interference that results in a loss of received power in a very localized area, a loss often referred to as fading. Furthermore, multipath can reduce the effectiveness of a channel by increasing inter-symbol interference. Here, a symbol is the fundamental unit of information. Although a signal may have a sufficient signal-to-noise ratio (SNR) at a receiving site, the signal may not be interpretable because it is composed of time-delayed replicas of the original transmission due to the multiple paths between transmitter and receiver.

Although not previously employed for communications systems, developments in Time Reversal Signal Processing (TRSP) indicate the potential for compensating the transmission channel while mitigating the need for detailed a priori knowledge of the channel characteristics. Furthermore its simplicity, viz. a viz. equalization, makes it particularly attractive. The successful use of TRSP can increase channel bandwidth, thereby enabling the proportional increase in the volume of information. It implicitly compensates for distortion by using the equivalent of an imbedded phase conjugation technique for the equalization. This is an astounding property of the TRSP that can be taken advantage of for this problem. This feasibility study is directed toward showing that TRSP is a viable method for mitigating the effects of multipath on the transmission of information through a communications channel.

In this report, we first briefly describe the theory behind the use of TRSP in communications. The channel is composed of a single transmitter-receiver pair operating in an unknown, possibly inhomogeneous, medium that includes scatterers that can contribute to multiple paths between the transmitter and receiver. These multiple paths

(multipath) express themselves as reverberation in the received signal and attendant distortion in the information. Once having established the theory behind the use of TRSP to mitigate multipath distortion, we will describe simulations of the performance of suggested systems using this approach. Finally, we will establish conclusions based on our observations in the simulations.

1.1 The Time Reversal Processor

Time Reversal Signal Processing has been the subject of research in recent years.^{1,2} The theory for applications in imaging has been developed and refined at LLNL. Recently there has been an extension proposed that will work for point-to-point communications. Conceptually, we have developed an approach based on the "time-reversal (TRP) processor" in a previous LDRD invention investigating the basic wave propagation properties and theory of the TRP for nondestructive evaluation.³ Time reversal is the dynamic *broadband* analog of the well-known phase conjugate mirror used to focus *narrowband* monochromatic waves. This same basic reversal principle holds in digital signal processing in two-pass digital filter design where a signal is filtered, reversed and re-filtered to provide an enhanced signal with the original phase preserved indicating a zero-phase filter response. In fact, from the signal processing perspective the TRP represents the "optimal" spatio-temporal matched filter in the sense of maximizing the output signal-to-noise ratio (SNR). It is essentially a technique, which can be used to "remove" the aberrations created by an inhomogeneous or random channel.

The basic principle of time reversal processing, in its simplest form can be succinctly characterized by the following. Consider the spatio-temporal propagation of source information, $i(r_o, t)$ located in space-time at r_o and t through a channel medium characterized by the channel Green's function (impulse response) $g(r, r_o; t)$. From systems theory we know that this operation is given by convolution to yield the received signal, that is,

$$R(r, t) = g(r, r_o; t) * i(r_o, t) \quad \xleftrightarrow{F} \quad R(r, \omega) = G(r, r_o; \omega) I(r_o, \omega), \quad (1)$$

where we have also shown both the time domain and frequency domain representations and their relationship through the Fourier transform. Based on the underlying theory, we "re-transmit" or "back-propagate" from r , through the channel to the original source position at r_o , the time-reversed signal, $R(r, -t)$, with the result

$$\hat{i}(r_o, t) = g(r_o, r; t) * R(r, -t) \quad \xleftrightarrow{F} \quad \hat{I}(r_o, \omega) = G(r_o, r; \omega) R^*(r, \omega), \quad (2)$$

where $*$ represents complex conjugation. Substituting the reversed signal into this equation and invoking reciprocity ($g(r_o, r; t) \equiv g(r, r_o; t)$) while interchanging source and receiver positions, we obtain

$$\hat{i}(r_o, t) = g(r_o, r; t) * g(r_o, r; -t) * i(r_o, -t) \quad \xleftrightarrow{F} \quad \hat{I}(r, \omega) = |G(r, r_o; \omega)|^2 I^*(r_o, \omega), \quad (3)$$

¹ Mathias Fink, "Time-Reversed Acoustics", Scientific American, Nov. 1999

² Mathias Fink and Claire Prada, "Acoustic time-reversal mirrors", Inverse Problems 17(2001) R1-R38.

³ James V. Candy, IL-10323

which implies that the reversed signals re-transmitted through the medium will recreate the original signal with some frequency dependent amplitude but without any phase change at the original source position.

We then have, with $R_{cc}(r_o, r; t)$ the temporal autocorrelation function of $g(r_o, r; t)$

$$\hat{i}(r_o, t) = R_{cc}(r_o, r; t) * i(r_o, -t) \quad \xleftrightarrow{F} \quad \hat{I}(r, \omega) = |G(r, r_o; \omega)|^2 I^*(r_o, \omega), \quad (4)$$

For narrowband signals where G is invariant with frequency and R_{cc} is impulsive, this is precisely phase conjugation. For wideband signals, the situation may be complicated by the lack of flatness in G .

Next we describe how the TR principle can be applied to the communications problem.

2. A Time Reversal Approach to Equalization in a Communications Channel

In the previous discussion of TRP we have employed a pitch and catch arrangement where a signal is moved first in one direction through the medium (pitch), received or caught, time reversed and sent back through the medium to the original location. For communications, the situation is slightly different. Here, the desired information signal must be transmitted from a source to a receiver using additional signals to aid in mitigating deleterious effects of multipath. In the following we describe the time reversal arrangement employed for enabling a channel.

One potential realization of the time reversal approach is achieved by first broadcasting a pilot signal through the channel from the receiver location to the source location. At the source location, the received pilot signal is time reversed and convolved with the information signal that is desired to be sent to the receiver. The resulting signal is transmitted from the source location to the receiver location. Mathematically, this realization of the TRSP approach is given below:

$$s(r_o; t) = g(r_o, r_r; t) * p(r_r; t) \quad (5)$$

where $s(r_o; t)$ is the signal collected at the source location r_o when a pilot signal $p(r_r; t)$ is sent from the receive location r_r . This signal is then reversed, convolved with the information signal $i(t)$ and transmitted through the channel where it is collected at the receiver location as:

$$q(r_r; t) = g(r_o, r_r; t) * s(r_o; -t) * i(t) = g(r_o, r_r; t) * [g(r_o, r_r; -t) * p(r_r; -t)] * i(t) \quad (6)$$

The effect of equalization is achieved by convolution of this received signal with the original pilot signal

$$e(r_r; t) = q(r_r; t) * p(r_r; t) = g(r_o, r_r; t) * g(r_o, r_r; -t) * p(r_r; -t) * p(r_r; t) * i(t) \quad (7)$$

or

$$e(r_r, t) = R_{gg}(r_o, r_r; t) * R_{pp}(r_r; t) * i(t) \quad (8)$$

with $R_{gg}(r_o, r_r; t)$ is the autocorrelation function of the channel Green's function and $R_{pp}(r_r; t)$ is the autocorrelation function for the pilot signal. The pilot signal is known since it was originally transmitted from the receiver location.

In the spectral or frequency domain the corresponding equivalents to Equations 7 and 8 are given by the following equations where we have made implicit use of *reciprocity*:

$$E(r_r; \omega) = G(r_o, r_r; \omega) G^*(r_o, r_r; \omega) P(r; \omega) P^*(r; \omega) I(\omega) \quad (9)$$

and

$$E(r_r; \omega) = |G(r_o, r_r; \omega)|^2 |P(r; \omega)|^2 I(\omega) \quad (10)$$

The same results can be obtained with all signal processing operations performed at the receiving location if both the source location and the receiving sites have knowledge of the pilot signal. In this case, the source first sends a pilot signal and then sends the information signal. At the receive location, the pilot is received and time reversed, then convolved with the received information signal, and the result convolved with the pilot signal that is known at both sites.

3. Establishing Feasibility Using Simulations

In order to test the feasibility of the communications scheme employing TRSP that was described in the previous section, a feasibility study was executed. First the theory was refined to assure completeness, then extensive sets of simulations were performed. The intent of the simulations was to provide an indication of the performance of the scheme and to establish some limits and bounds on that performance.

3.1 The Postulated Problem and the Propagation Environment

As a test case for the TRSP communication system, it was decided that a bi-phase modulated carrier, i.e., a bi-level (+1, -1) code stream modulating a carrier would serve as an information signal. This signal would then be propagated through an environment capable of introducing severe multipathing effects as well as white noise into the signal stream. The scattering environment causing the multipaths is composed of randomly located scatterers, large in number N , and scattering as perfect mirrors.

Figure 1 is a graphical representation of a system that includes both the time reversal scheme and the classical equalization scheme for recovering the information from a signal that has propagated through the degrading medium. The TRSP approach is represented in the upper right hand portion for the figure while classical equalization is represented in the lower right hand portion. In either case, two signals are transmitted i.e., a pilot signal (or sequence) and an information signal (or sequence). For the purposes of our simulations, both sequences are generated using pseudo-random noise (PRN) with different PRN sequences used for the pilot and information. Also shown in

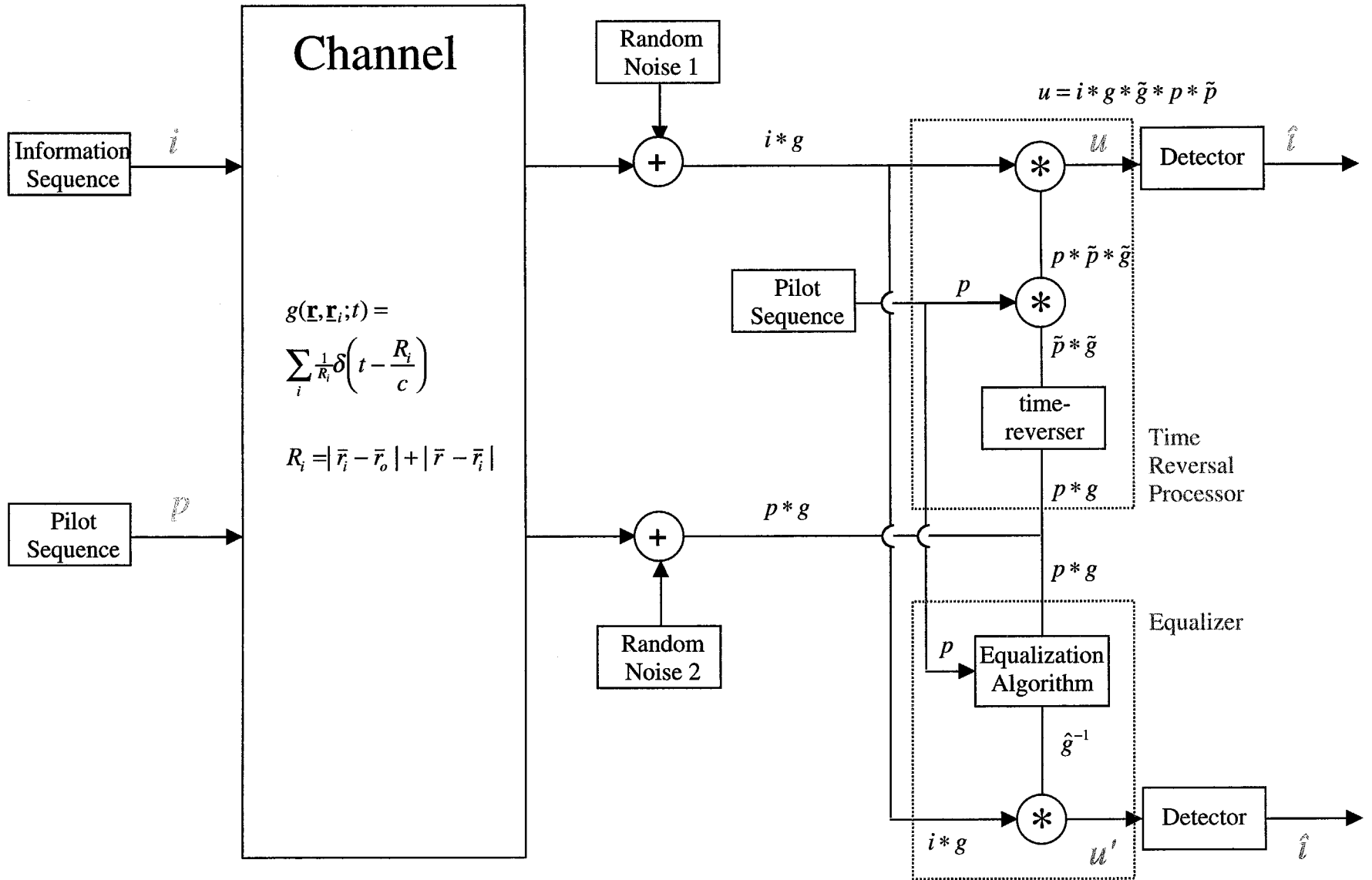


Figure 1. Graphical representation of a communication system including both the TRSP and classical equalization approaches. This shows the elements of the entire system including the propagation medium, the information and pilot signals, and the processing used in the receivers.

the figure are the signals (without the explicit time variable) that exist in each branch. The signal representations do not include contributions from the random noise. The origin of these signals is described in following paragraphs. It should be noted the tilde (\sim) symbol represents a time-reversed signal and the caret (\wedge) symbol represents an estimate of the variable.

The medium is assumed to be a typical communication environment composed of a uniform, homogeneous propagation medium in which are embedded many (N) scatterers. The Green's function for a source at r_0, t_0 in a homogeneous medium as shown in Figure 2 is

$$g(\bar{r}, \bar{r}_o; t, t_o) = \frac{1}{R} \delta(t - t_o - R/c) \quad (11)$$

$$R = |\bar{r} - \bar{r}_o|$$

We chose to represent the scattering phenomenon as that due to a perfect mirror thus merely introducing a time shift and amplitude reduction that depends on the total length of the path from transmitter to scatterer $|\bar{r}_i - \bar{r}_o|$ and the scatterer to the receiver $|\bar{r} - \bar{r}_i|$ (see Figure 2). Thus the Green's function for propagation in the medium is

$$g(\bar{r}, \bar{r}_o; t, t_o) = \sum_{i=1}^N \frac{1}{R_i} \delta(t - t_o - R_i/c) \quad (12)$$

$$R_i = |\bar{r}_i - \bar{r}_o| + |\bar{r} - \bar{r}_i| = r_{i1} + r_{i2}$$

For our simulations, the direct path from transmitter to receiver without scattering was numbered $i=1$ with $R_1 = |\bar{r} - \bar{r}_o|$.

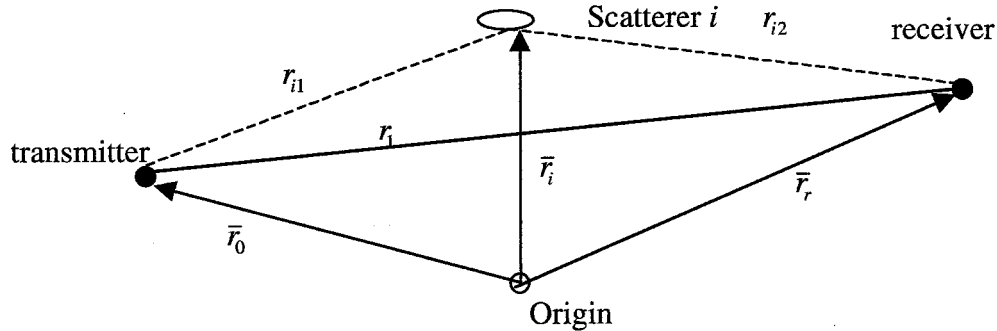


Figure 2. Geometry for scatterers causing multipath

The environment is capable of introducing noise into the system. We have chosen such noise to be additive white noise with different realizations being introduced for the pilot and information portions, since they occur at different times during our transmissions.

3.2 The Processors

As described before, Figure 1 shows elements of the entire simulation structure for our feasibility study including both the TRSP processor and the classical equalization approach.

The equalization algorithm used in this study can be understood by modeling our propagation problem as a linear system. Figure 3 is the system with impulse response $g(t)$ that is our Green's function given by

$$y(t) = g(t) * x(t) = \sum_{k=0}^{M-1} g(k)x(t-k) = [g(0) \dots g(M-1)] \begin{bmatrix} x(t) \\ \vdots \\ x(t-M+1) \end{bmatrix} = \underline{g}^T \underline{x}(t) \quad (13)$$

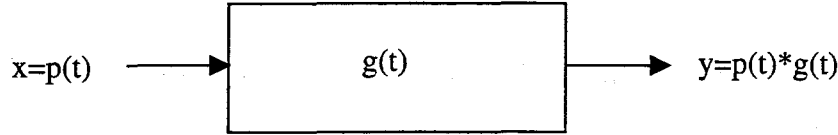


Figure 3. A systems representation

Here we estimate the Green's function using a standard Wiener solution with all system effects included in the time domain as $\hat{\underline{g}}$, as in

$$\hat{\underline{g}} = R_{xx}^{-1} r_{yx} \quad (14)$$

with R_{xx} an $M \times M$ correlation matrix (Toeplitz) and r_{yx} a M -cross-correlation vector. Also, due to the commutivity of the convolution operator, the inverse filter is estimated in the same way as

$$\hat{\underline{g}}^{-1} = R_{yy}^{-1} r_{xy} \quad (15)$$

A recursive (in time) algorithm can be applied to solve this problem efficiently (order M^2 rather than M^3). It is called the Levinson-Wiggins-Robinson (LWR) recursion.

The detector used in the system was very simply a signum detector that quantizes to +1 and -1 depending on whether the variable is positive or negative. Given that the transmitted information sequence was bi-level, i.e., +1 or -1, and that the information signal was bi-level at baseband (no carrier) or bi-phase, i.e., 0 or π radians in phase of the carrier for operation at frequency, a perfect system would yield signals at the system output that were either +1 or -1 after any demodulation of the signals (if required). Most simple receivers would use such a detector. We would thus have the following estimates:

$$\hat{i}(t) = \begin{cases} +1 & u(t) \geq 0 \\ -1 & u(t) < 0 \end{cases} \quad (16)$$

where $u(t)$ is the processed signal prior to detection as shown in Figure 1.

4. The Digital Implementation for Simulations

4.1 Randomly Located Scatterers

In order to enable simulations to assess system performance, the system shown in Figure 1 was simulated. We have already stated that performance of the TRSP approach was optimal when R_{gg} in Equation (8) is impulsive or equivalently when $|G(r_0, r_r; \omega)|$ is flat or constant as a function of frequency. This follows from the fact that the pilot signal, autocorrelation and spectrum are known, so that $i(t)$ and $I(\omega)$ can be recovered precisely from either $e(r_r, t)$ and $E(r_r, \omega)$. We know that the autocorrelation is impulsive for white noise. We have sought to emulate such a condition in our system by postulating a large number of scatterers that are randomly located in a homogeneous space. Doing this, we expect the autocorrelation of g as given by Equation (12) to be impulse-like, i.e., small except for zero lag.

If the scatterers in our homogeneous space are randomly located, then the respective path lengths R_i and amplitude associated with each path $1/R_i$ are expected to also be random. We created such random lengths and amplitudes by:

- 1) Choosing the number (N) of scatterers in our system.
- 2) Creating a pseudo-random number sequence of length $M > N$. The PRN is obtained by generating a random permutation of numbers between 1 and M . Note that there are no repetitions in such a sequence.
- 3) In order to allow for as many as two path lengths being identical, we created two PRN sequences of length M , ordered each and chose the first $N/2$ from each sequence to form a new sequence of length N .
- 4) The new sequence was ordered and the set of path length and associated intensity given by $\left\{ R_i, \frac{1}{R_i^2} \right\}$ was created for the index $i \in (1, N)$.

4.2 The Pilot and Information Sequences

If the pilot signal were an impulse $\delta(t)$, then we see that $g(r, r_0; t)$ is obtained immediately as can be inferred from Equation (5). Also, if it were pure white noise, the spectral representation of g could be obtained exactly. Neither situation being possible, we chose to create a pilot sequence from a PRN sequence. For the purposes of this study, we also chose the information sequence to be a PRN sequence. The sequences were created as follows:

- 1) Choose a symbol rate R_s appropriate for the information or pilot desired to be transmitted. This rate implies a chip rate $\tau = 1/R_s$. For the time length of the signal T , we then have a total of $K = T R_s$ chips or symbols.
- 2) Randomly permute a sequence from 1 to K to obtain a PRN sequence on the interval $(1, K)$.
- 3) Subtract the mean value, $(K+1)/2$, from each entry yielding a sequence on the interval $(-K/2-1/2, K/2-1/2) \sim (-K/2, K/2)$ for large values of K .

- 4) Apply the signum function to obtain a pseudo random sequence of the symbols +1 and -1.

Clearly each individual pulse in the train or chip of length τ has an associated bandwidth given by $2/\tau$. Since the pilot signal effectively probes or characterizes the medium, its bandwidth must exceed that of the information signal. For our study, we chose the bandwidth of the pilot signal to be an order of magnitude larger than the bandwidth of the information signal.

4.3 Operation at Baseband and at Frequency

During the course of our study we performed simulations of the system both at baseband and at frequency, as previously mentioned. Here we describe how this was achieved in a digital framework. It is pointed out at the outset that all time reversal signal processing was performed at the baseband frequencies. In essence, the carrier was introduced to provide a signal capable of propagation through a medium. One might ask Why bother dealing with the at frequency transmission using a carrier if all signal processing is done at baseband? We deal with these aspects because the modulation-demodulation steps can introduce noise and distortions in the signals that might compromise the process.

Throughout we refer to the information sequence $i(t)$ and pilot sequences $p(t)$ as the data streams that serve as the input to the communication channel. They are composed of sets of chips that form symbols. When we introduce a carrier, we merely modulate these sequences by the sinusoidal carrier to obtain, $i(t)\sin\omega t$ and $p(t)\sin\omega t$.

The modulation process can produce spectral components at frequencies sufficiently removed from the carrier. To suppress these effects, we use a bandpass filter around the carrier frequency. The width of this passband is chosen to be several times the bandwidth of the modulating signal. We represent this modulation operation schematically as shown in Figure 4.

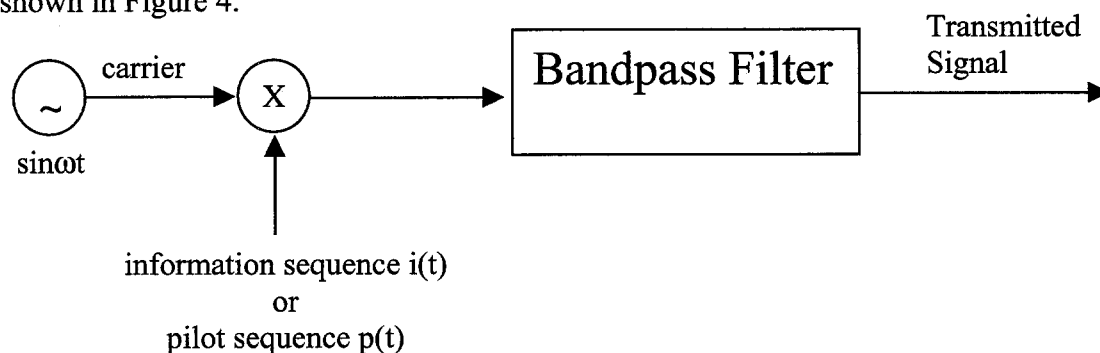


Figure 4. Creating the transmitted signal for the at frequency case

At the receiving end, the reverse process is carried out. There the carrier is removed by performing a demodulation. This process is depicted in Figure 5.

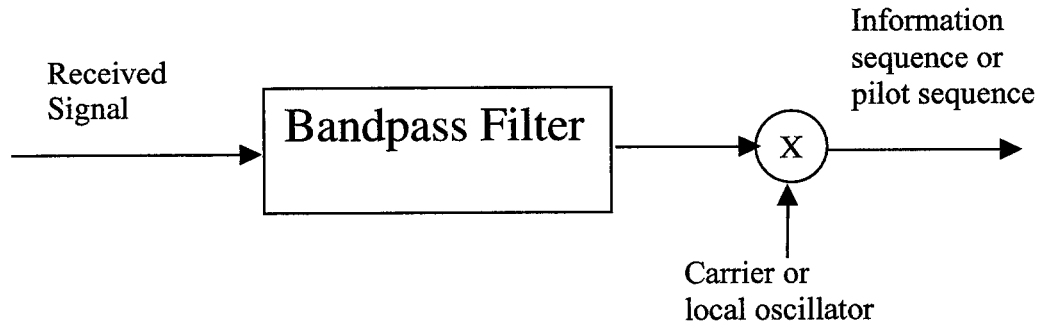


Figure 5. Demodulating the received signal to extract the desired sequences

5. The Simulations

5.1 Demonstration of the Process

A series of simulations were executed to determine the feasibility of TRSP in communications and to compare the method to classical equalization. The parameters for these tests are shown in Table 1.

Table 1. Parameters for System Simulations

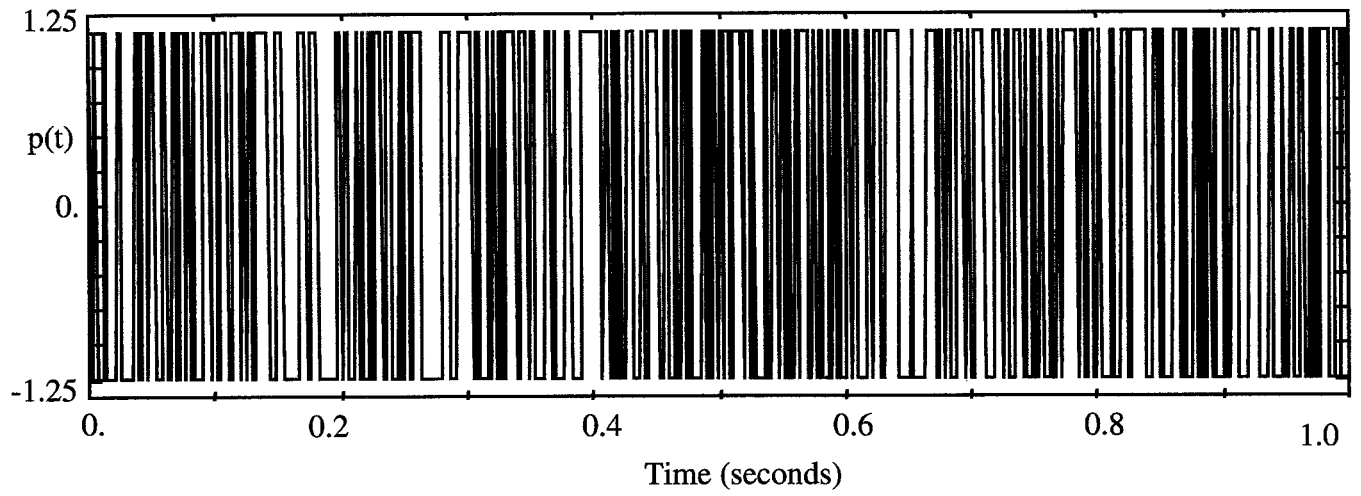
	Pilot	Information
Bandwidth	400 Hz	40 Hz
Chip Length	5 ms	50 ms
Sequence Length	1 s	1 s
Number of Chips	400	40

For the case where a carrier was present, the carrier frequency was arbitrarily chosen to be 800 Hz.

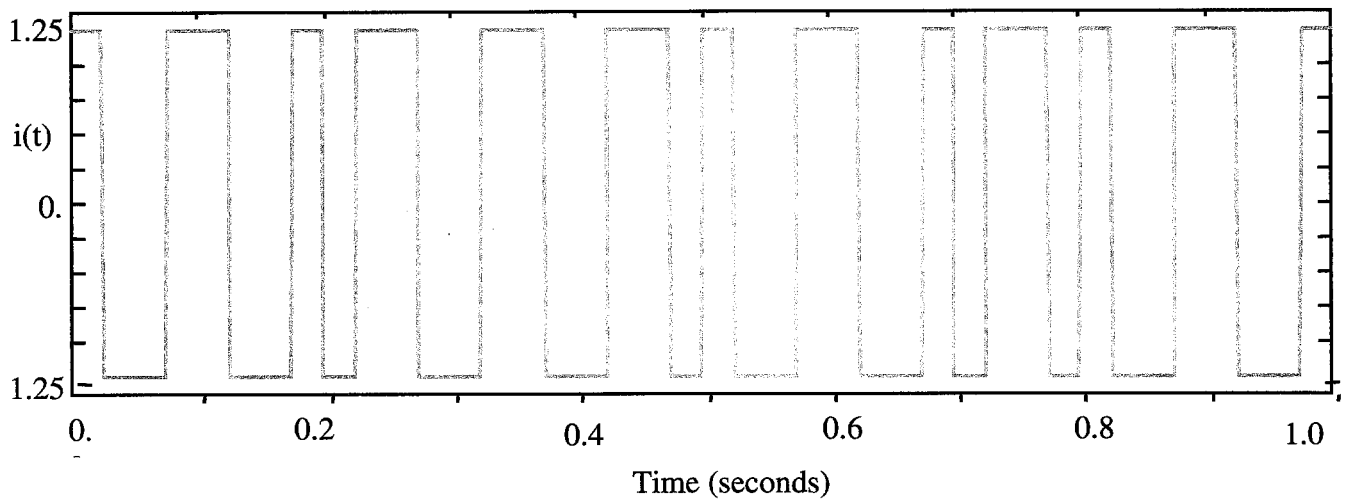
The PRN sequences were created using a generator in MATLAB. For every different case in our study a different realization of the PRNs was used. But, the seed for the generator for each realization was retained so that the computations and processing for any particular realization could be repeated at a later time, if necessary.

Simulations were performed using the parameters in Table 1. For illustrative purposes the results for a single realization of the PRNs for pilot and information are shown in Figures 6 through 10. The intent is to illustrate the performance of the TRSP processor.

In Figures 6a and b the sequences for the pilot and information are shown, respectively. Remember that the bandwidth of the pilot is ten times that of the information for reasons discussed earlier.



a) The pilot sequence



b) The information sequence

Figure 6. The pilot and information sequences for the simulations

Figure 7 is a plot of the information signal after it has propagated through the medium with a typical number of scatterers. It is referred to as $z(t)$ where $z(t)=i(t)*g(r;t)=i*g$ in the shorthand notation of Figure 1. The degraded character of the signal is evident.

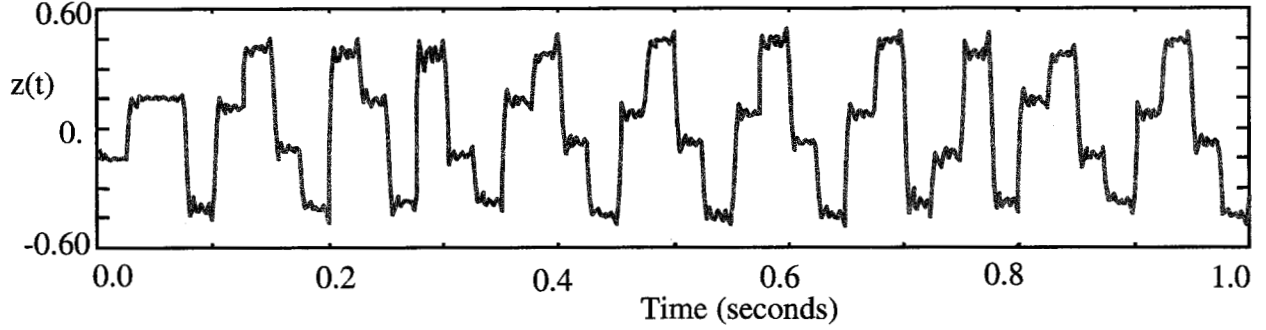


Figure 7. The information signal after propagation through the medium

Figure 8 is the estimate of the information signal in the receiver after it has been processed by a detector that employs a signum function. This estimate is referred to as $Est(i_g)$. In the figure, the red curve is the transmitted sequence while the blue curve is the estimate at the output of the detector. Significant errors are evident in the estimate.

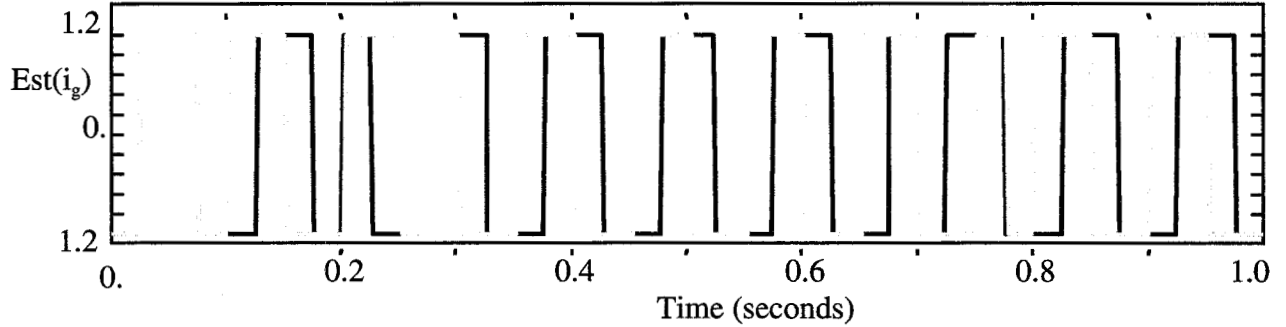


Figure 8. An estimate of the information sequence after detection using the signum function. The red curve is the transmitted function and the blue curve is the estimate. The differences and subsequent errors are evident.

Figure 9 is a plot of the received signal from Figure 7 after it has been processed using time reversal methods. This signal is identified in Figure 1 as the signal just prior to detection in the TRSP part of the processor and is given by

$$u(t) = i(t) * g(r,t) * g(r,-t) * p(t) * p(-t) = i(t) * R_{gs}(r,t) * R_{pp}(t) \quad (17)$$

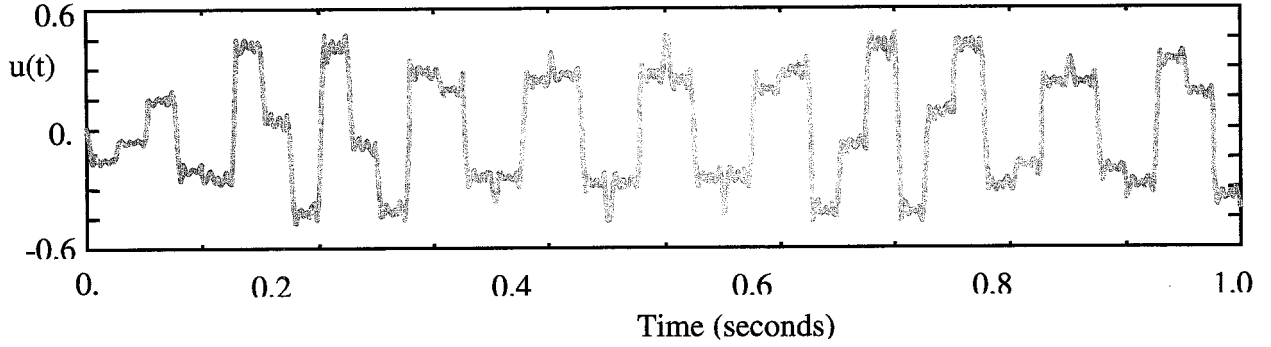


Figure 9. The information signal after time reversal processing but before detection

Figure 10 is a plot of the estimate of the information sequence $\hat{i}(t)$ that is obtained by applying a detection process to the signal $u(t)$ in Figure 9. The detector employs a signum function. Note that in this processing we have not used any assumptions regarding any of the functions.

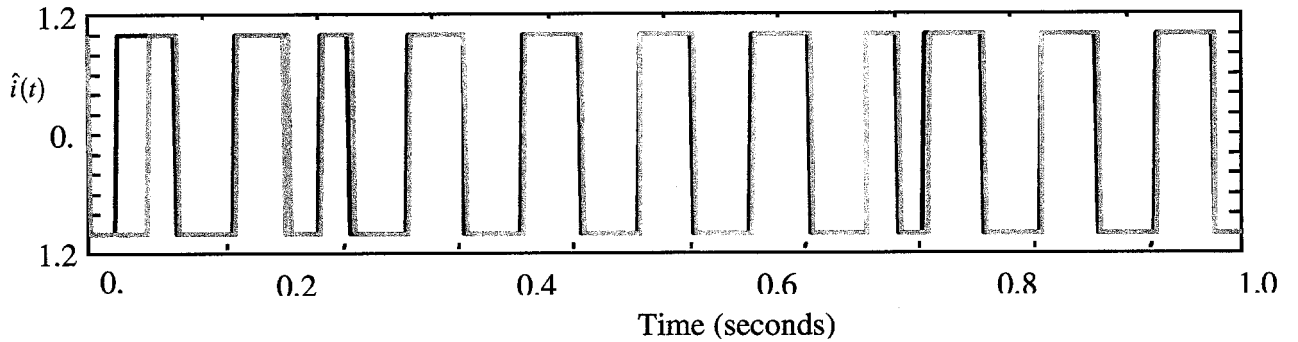


Figure 10. An estimate of the information sequence after detection

The preceeding demonstration has shown that the TRSP-based processing scheme has promise in communications. We will now proceed to evaluate its performance on a broader scale and compare it to a classical equalization scheme.

5.2 Evaluation of TRSP Performance

The performance of the TRSP process has been carried out using simulations. As a point of comparison, a classical equalization method has been applied to the same data. The variables in the comparison, which have been shown to be most important in measuring performance, are the signal-to-noise ratios of the signals available for processing and the number of scatterers or multipaths in the simulation.

We expected that the results that would be obtained with both approaches would be dependent on the number of scatterers (also known as density of multipaths) and the particular forms of the pilot and information signals, i.e., the particular realizations of each. So, during the course of our simulations we first chose the multipath density and performed simulations using both TRSP and classical equalization.

An error metric was established which would allow comparison of the methods. For each realization of pilot and information sequence, the samples of the estimated information sequence at the output was compared to the input sequence. The squared error was evaluated and averaged over the number of samples in the sequence. For the j^{th} realization of pilot and information sequences we have

$$E_j = \frac{1}{Q} \sum_{k=1}^Q |\hat{i}_k - i_k| \quad (18)$$

where subscript k refers to the k^{th} sample in the time sequence for the signal i . The number of samples is chosen to be much larger than the number of chips under consideration so that each chip is sampled several times. This particular metric was evaluated for many realizations of the pilot and information sequences for a given multipath density (N) so that a mean value referred to as Error and variance could be established. We evaluated the mean as:

$$Error = \langle E_j \rangle^{1/2} = \left(\frac{1}{N} \sum_{j=1}^N E_j \right)^{1/2} \quad (19)$$

Another variable which could impact performance was the signal-to-noise ratio (SNR) of the data. During the course of our study we routinely used an SNR range from 1000 dB (effectively noise-free) to 2 dB which corresponds to noise power that is 63% as large as the signal power, a rather substantial noise level.

For the purpose of comparison, a classical equalization was also used in the estimation of the information sequence as shown in Figure 1. Ordinarily, one would hope to perform the operation indicated in Equations (13) and (14) for an entire signal stream at the receiver due to transmission of the pilot signal. This would include the effects of all scatterers on signal propagated in the medium and would likely lead to the best estimate or least error for that particular realization of the propagation channel. However, this is extremely intensive in terms of computational requirements from the standpoint of computational effort in dealing with large data sets and the fact that it may be an ill-posed problem. In all likelihood such an approach would not be used. In view of these difficulties, we chose to perform an equalization using only the direct path from transmitter to receiver, i.e., the equalization of the channel was performed for the case of no multipath. This equalizer was then used to try to achieve equalization for the multipath problem.

Figures 11 through 13 are condensations of the results of our simulations. They are plots of our metric (Equation (18)) versus density of multipath or number of multipaths.

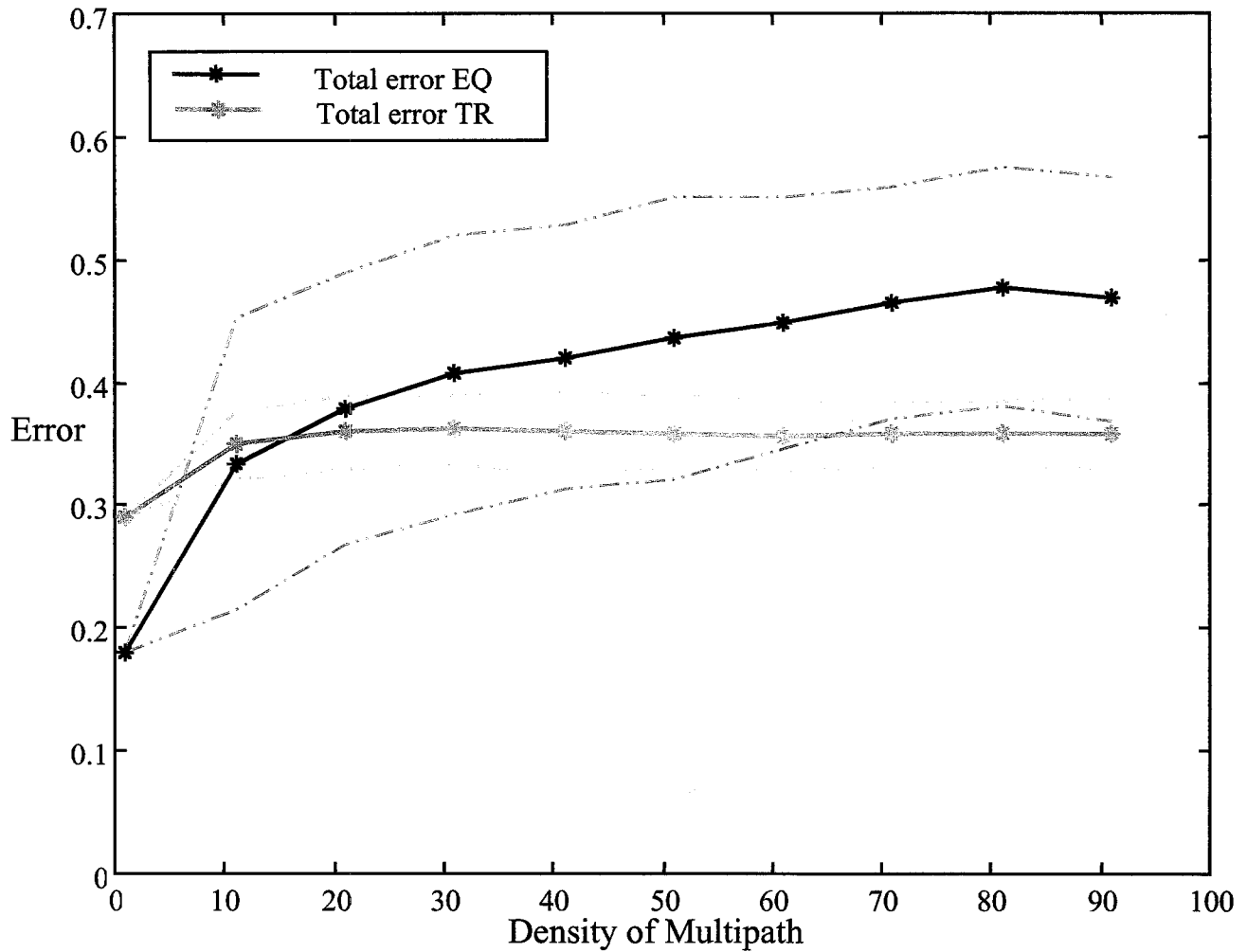


Figure 11. Comprison of Error using Equalization and TRSP versus multipath density for a SNR = 1000 dB (effectively noise-free). The dashed lines represent the 1σ variance bounds for the respective cases. All processing was at baseband.

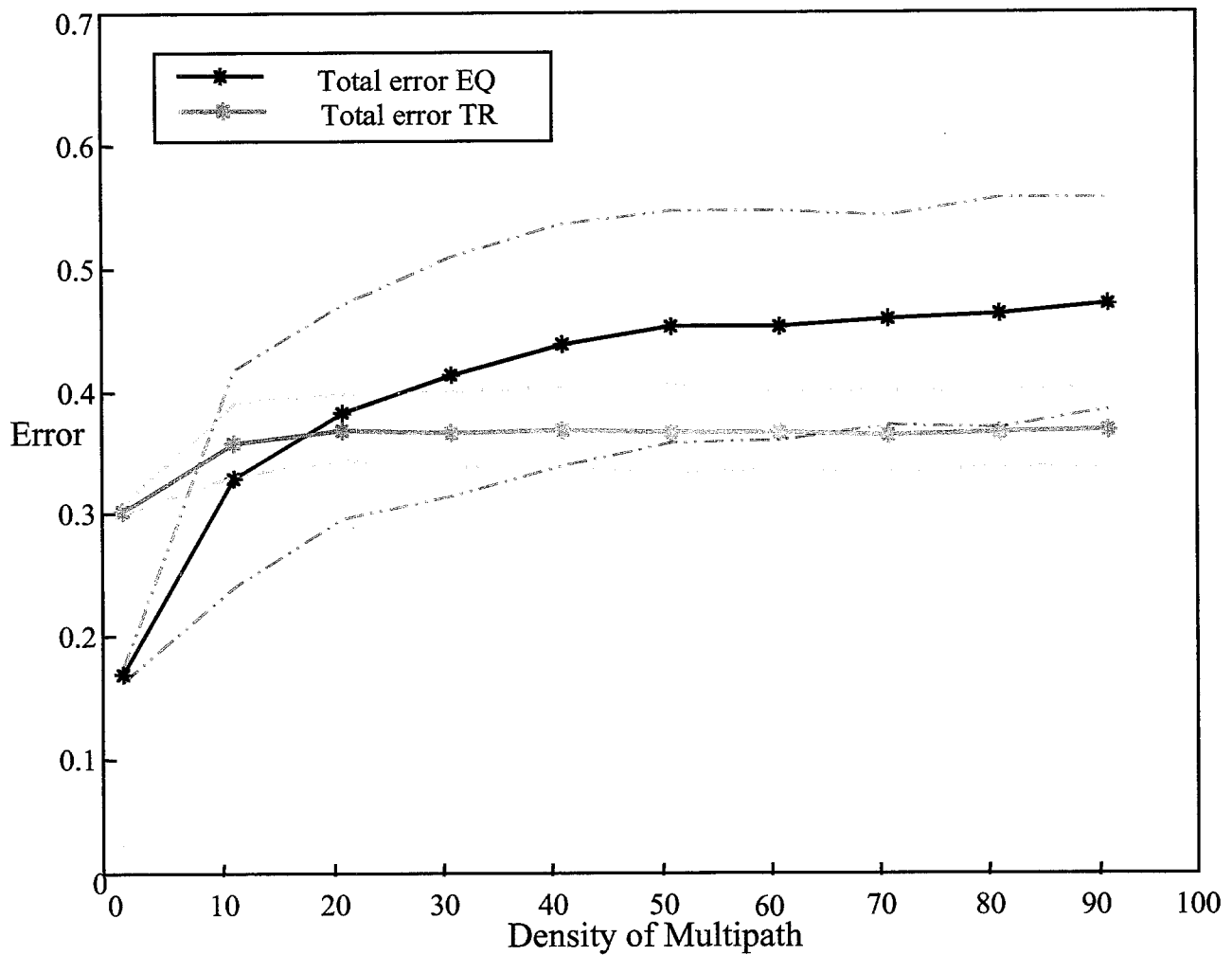


Figure 12. Comprison of Error using Equalization and TRSP versus multipath density for a SNR = 25 dB. The dashed lines represent the 1σ variance bounds for the respective cases. All processing was at baseband.

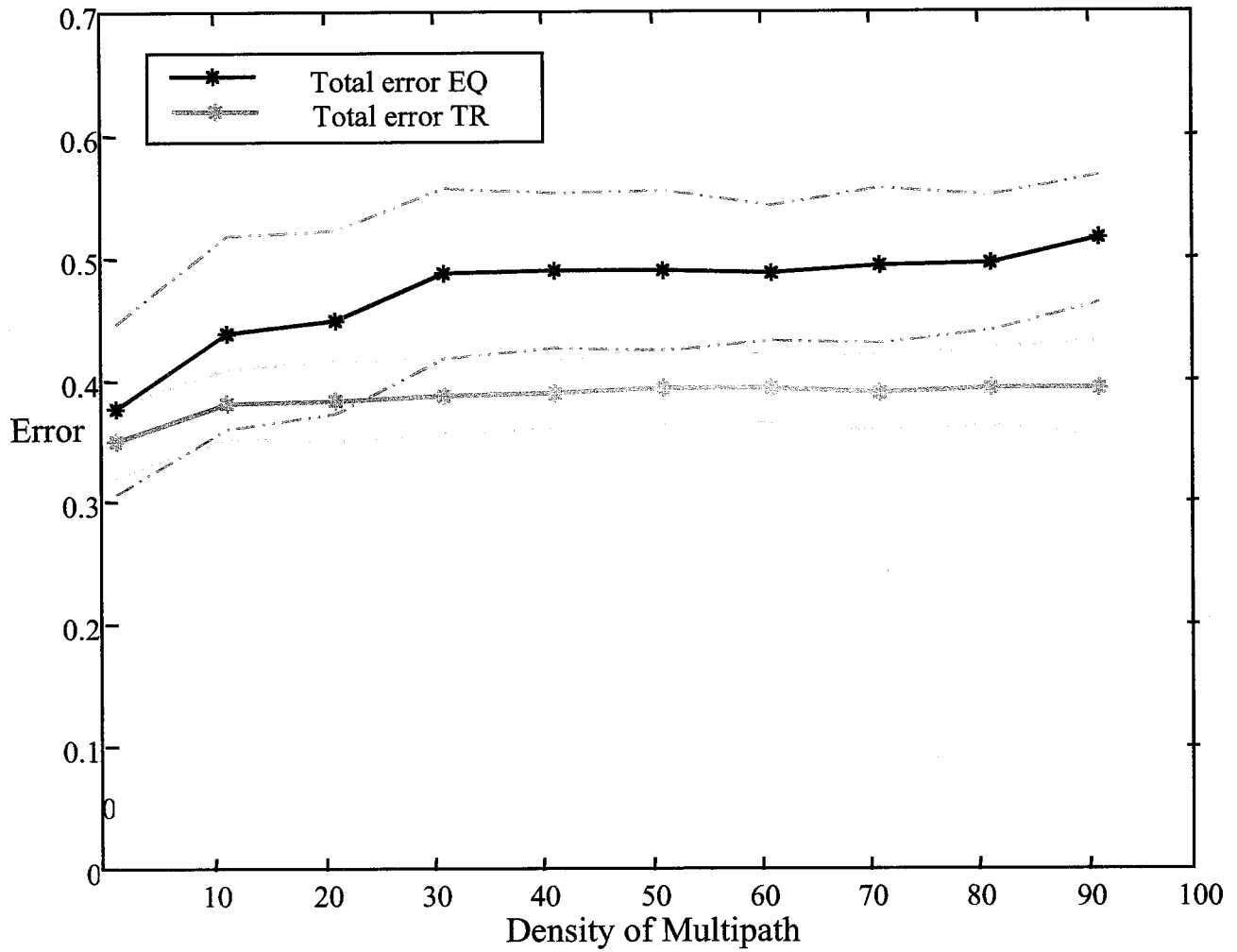


Figure 13. Comprison of Error using Equalization and TRSP versus multipath density for a SNR = 2 dB. The dashed lines represent the 1σ variance bounds for the respective cases. All processing was at baseband.

It is evident from the summary curves in Figures 11 through 13 that the TRSP system is stable with respect to increasing numbers of scatterers and has a uniform error after the system includes just a few scatterers. On the other hand, the equalization approach seems to have an error that increases with the density of multipaths. We also note that the error using equalization is smaller than for TRSP at low numbers of multipaths. One must remember that the parameters of the equalizer were established using a signal that was direct, i.e., there were no multipath components in the signal. In reality, the received signal was truncated before any time delayed components were received. Note that for long responses this could be difficult to achieve in reality. Also note that once equalized using a signal, our system performance could be perfect for an exactly similar situation. However, also note that the system can experience degradation after one leaves this perfect situation. In fact, a test was performed where the equalizer parameters were set in the presence of 20 multipaths. Then a complete evaluation was performed and the resulting Error vs. Density plot had a deep notch at 20 and severely degraded performance elsewhere.

Another observation in Figures 11 through 13 is that the variance in performance for the TRSP approach is smaller than for the equalizer approach. This is indicative of a tighter bound on expected performance for the TRSP system.

Some computational experiments were performed to establish whether the performance of the TRSP system was sensitive to whether the modulation and demodulation processes that were required to prepare an information sequence for transmission would degrade the process. Figure 14 is a test case that was executed without noise but with all processing functions performed. This test demonstrated that the introduction of a carrier did not degrade the system performance.

6. Conclusions

Based on the simulations executed during this feasibility study the Time Reversal Signal Processing approach in communications has been shown to be an excellent candidate for mitigating the effects of severe multipath. Using rather straightforward operations on the signals, it has been shown to compete with the performance of equalization schemes especially in the presence of a large number of scatterers causing multipath. It has also been shown to be robust in the presence of noise. The TRSP approach in communications has been observed to require rather straightforward signal processing operations that are robust and straightforward. The observations in this project warrant an experimental program that would test the system in a realistic environment.

It should be noted that time reversal is not limited to point-to-point communications. In fact, using an array of transmitters or receivers enables the T/R array to focus on the source directly which could be very important for secure communications in a noisy, highly reverberant environment.

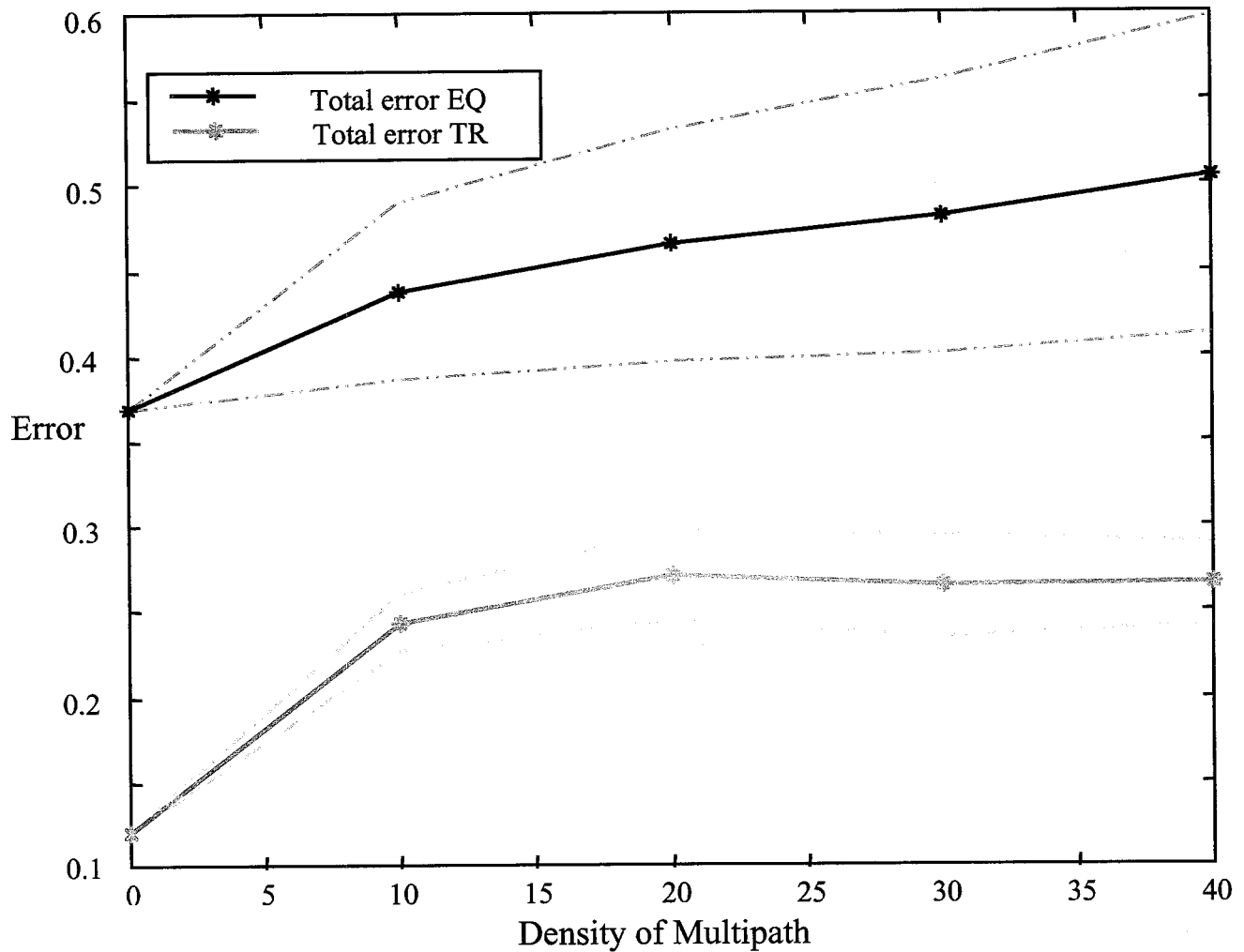


Figure 14. Comprison of Error using Equalization and TRSP versus multipath density for a SNR = 1000 dB (effectively noise-free). The dashed lines represent the 1 σ variance bounds for the respective cases. Signal transmission through the medium with scatterers was performed at a carrier frequency of 800 Hz.